

# Comparison of two non-linear model-based control strategies for autonomous vehicles

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# Outline

1. Motivation
2. Introduction
3. Control Oriented Vehicle Model
4. Two non linear control strategies
5. Simulation Results
6. Experimental Results
7. Conclusions

# 1. Motivation

# 1. MOTIVATION

Recent advances in C3 (Control-Computation-Communications) have made it possible to develop autonomous vehicles that exhibit a high degree of reliability in their operation, in the face of dynamic and uncertain environments, operating conditions, and goals.

Autonomous driving has been an important topic of research in recent years and numerous major companies and research organizations have developed working prototype autonomous vehicles (Mercedes-Benz, General Motors, Continental Automotive Systems, IAV, Autoliv Inc., Bosch, Nissan, Renault, Toyota, Audi, Volvo, Tesla Motors, Peugeot, AKKA Technologies, Vislab from University of Parma, Oxford University and Google)

## 2. Introduction

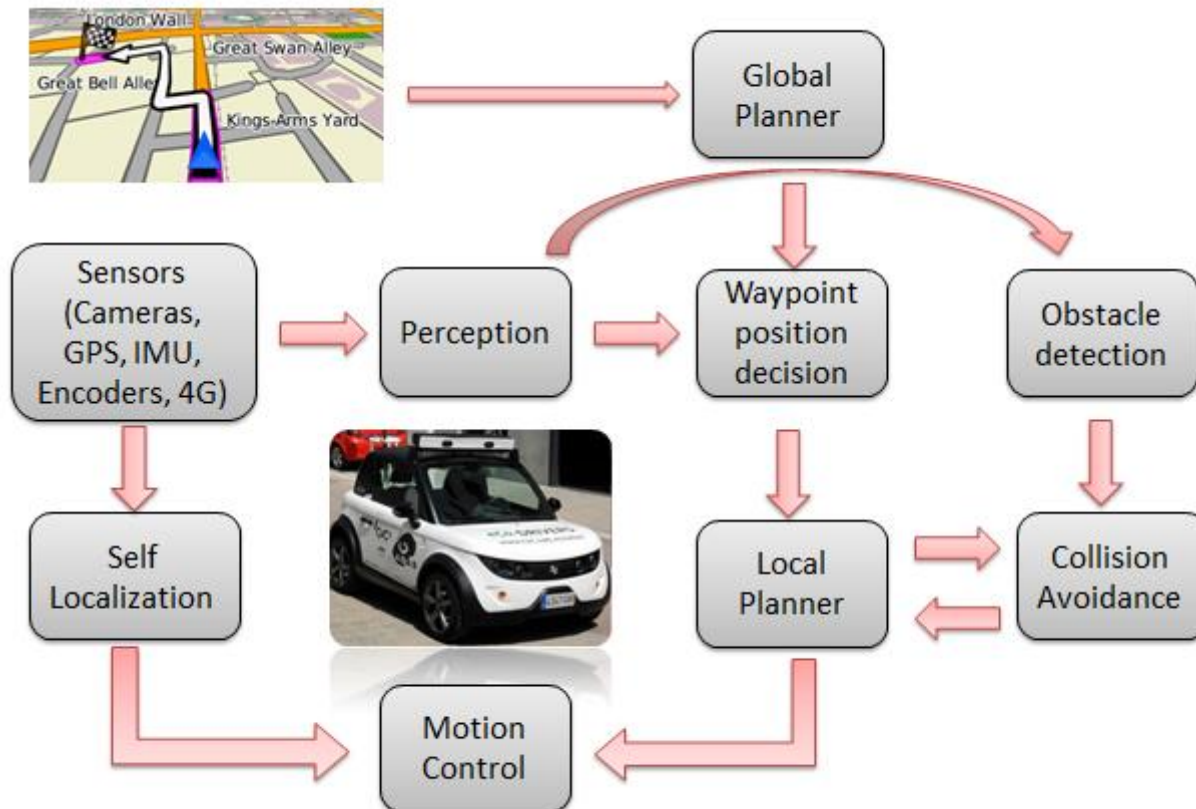
## 2. INTRODUCTION

The Computer Vision Center (CVC) is automatizing an electric car within the context of the project Automated and Cooperative Driving in the City (ACDC)



## 2. INTRODUCTION

In particular, while following a planned route, the obstacle-free navigable path in front of the vehicle is detected by using an on-board stereo rig. Accordingly, a short path is planned obtaining the desired set of positions and velocities. Such a set is sent to the car controller to properly execute the maneuver:



## 2. INTRODUCTION

This paper is focused on the low frame of the automatic control of the speed and the steering angle of the car following a predefined path with the best performances of stability and precision.

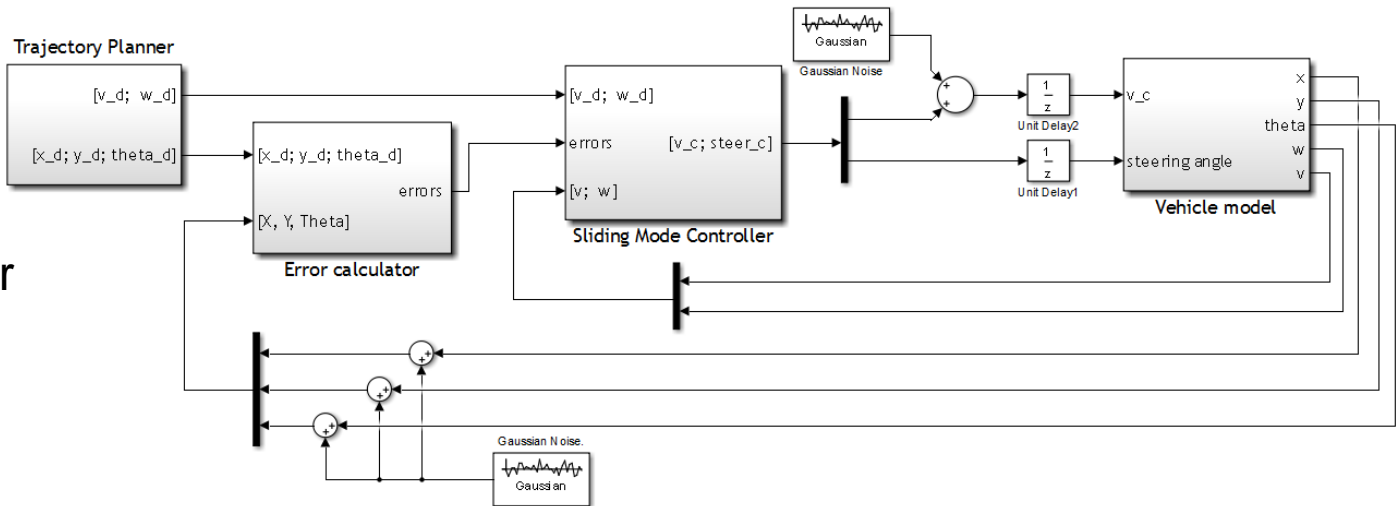
We propose to apply two strategies of non-linear automatic low level control, based on the method of Lyapunov proposed by (Aicardi, 1995) and based on Sliding Model Control (Gao,1993).

And a comparison of both has been made in a simple simulator (based on Simulink) and tested in a complex simulator developed in Unity 3D2.

Currently, it is being tested in the real autonomous car.



# 2. INTRODUCTION



Simulink simulator

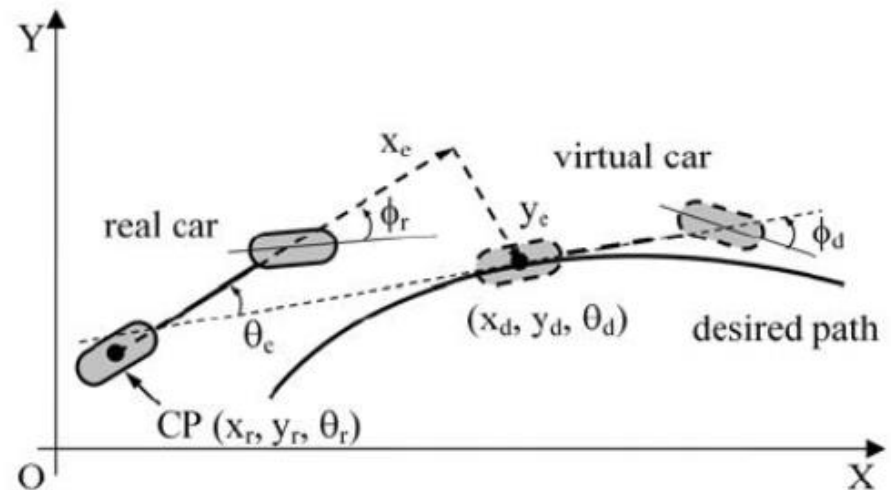
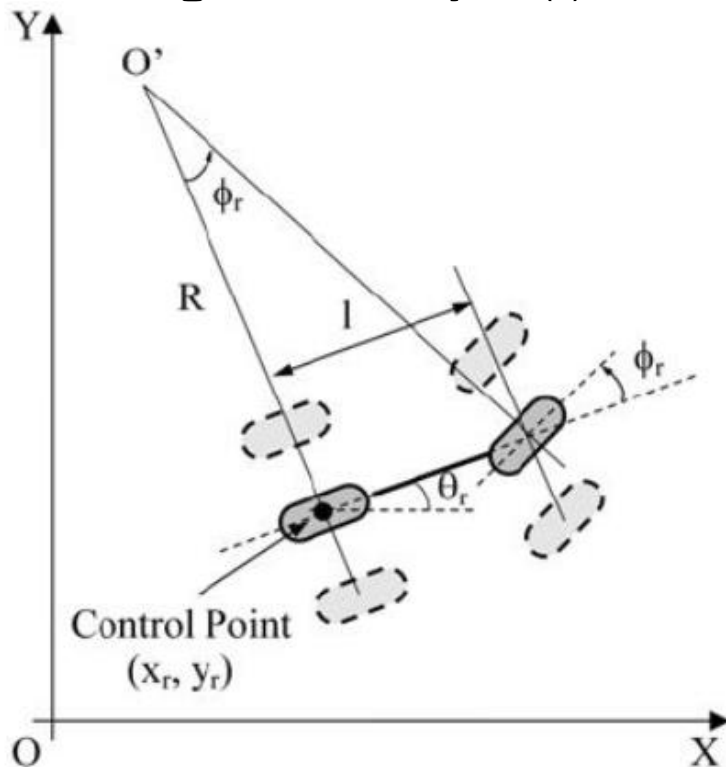


Unity simulator

### **3. Control Oriented Vehicle Model**

### 3. CONTROL ORIENTED VEHICLE MODEL

For control design, the autonomous car has been considered as a bicycle-like vehicle positioned at a nonzero distance with respect to a dynamic waypoint (virtual car of reference), whose motion is controlled by the combined action of both the angular velocity  $w_r(t)$  and the linear velocity  $v_r(t)$  of the real vehicle.

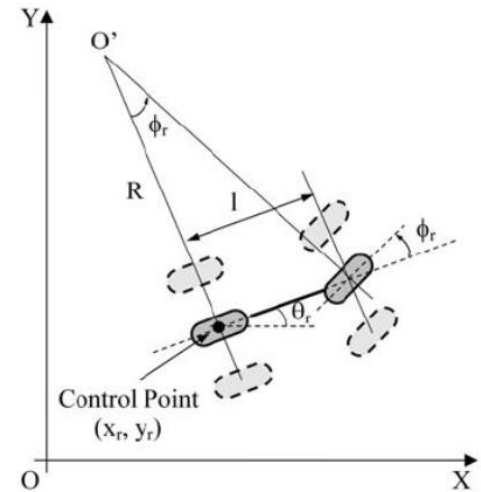


# 3. CONTROL ORIENTED VEHICLE MODEL

Then, the set of kinematic equations of the cartesian position  $(x_r, y_r)$  and orientation  $(\theta_r)$  of the real vehicle is presented as follows:

$$\begin{cases} \dot{x}_r = v_r \sin(\theta_r) \\ \dot{y}_r = v_r \cos(\theta_r) \\ \dot{\theta}_r = \frac{v_r}{l} \tan(\phi_r) \end{cases} \quad (1)$$

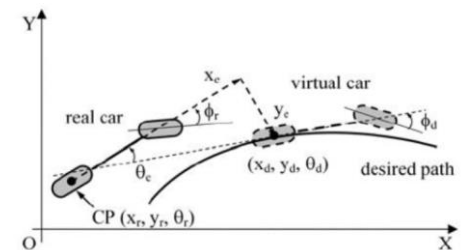
where  $v_r$  and  $\phi_r$  represent the linear velocity and the steering angle respectively.



the kinematic equations for the virtual car can be defined as:

$$\begin{cases} \dot{x}_d = v_d \sin(\theta_d) \\ \dot{y}_d = v_d \cos(\theta_d) \\ \dot{\theta}_d = \frac{v_d}{l} \tan(\phi_d) \end{cases} \quad (2)$$

where  $x_d$ ,  $y_d$  and  $\theta_d$  are the position and orientation of the next way point generated by the trajectory planner.



### 3. CONTROL ORIENTED VEHICLE MODEL

The error model is defined as the difference between real vehicle position and the desired one multiplied by the rotation matrix over  $z$  axis which is the orthogonal to the road plane:

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos\theta_d & \sin\theta_d & 0 \\ -\sin\theta_d & \cos\theta_d & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_d \\ y_r - y_d \\ \theta_d - \theta_r \end{bmatrix}$$

that after some algebraic manipulations lead to the following expression:

$$\begin{cases} \dot{x}_e = v_r + \omega_r y_e - v_d \cos(\theta_e) \\ \dot{y}_e = -\omega_r x_e + v_d \sin(\theta_e) \\ \dot{\theta}_e = \omega_r - \omega_d \end{cases}$$

## **4.- Two non linear control strategies**

# 4. TWO NON LINEAR CONTROL STRATEGIES

## A. Direct Lyapunov approach

We are going to design a control based on the direct Lyapunov approach. This method guaranties the asymptotic stability of the vehicle control because:

$$\lim_{t \rightarrow \infty} \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = 0$$

which involves also:

$$\lim_{t \rightarrow \infty} \begin{bmatrix} x_r - x_d \\ y_r - y_d \\ \theta_r - \theta_r \end{bmatrix} = 0$$

As control law we propose to use the non linear law from

$$\begin{bmatrix} v_r \\ \omega_r \end{bmatrix} = \begin{bmatrix} v_d \cos \theta_e - k_1 x_e \\ \omega_d - k_2 v_d \frac{\sin \theta_e}{\theta_e} y_e - k_3 \theta_e \end{bmatrix}$$

Given the following Lyapunov function:

$$V = \frac{1}{2} x_e^2 + \frac{1}{2} y_e^2 + \frac{1}{2} \theta_e^2$$

the stability condition is achieved when  $\dot{V} \leq 0$ .

$$\dot{V} = -k_1 k_2 x_e^2 - k_3 \theta_e^2 \leq 0$$

which implies that the control parameters  $k_1$ ,  $k_2$  and  $k_3$  should be positive to assure the asymptotic stability of the closed-loop.

# 4. TWO NON LINEAR CONTROL STRATEGIES

## B. The Sliding Control Approach

The main idea behind this approach is to reach the sliding surface in a finite time and remain on such surfaces where the error is null.

The resulting surfaces are the following:

$$s_1 = \dot{x}_e + k_1 x_e$$

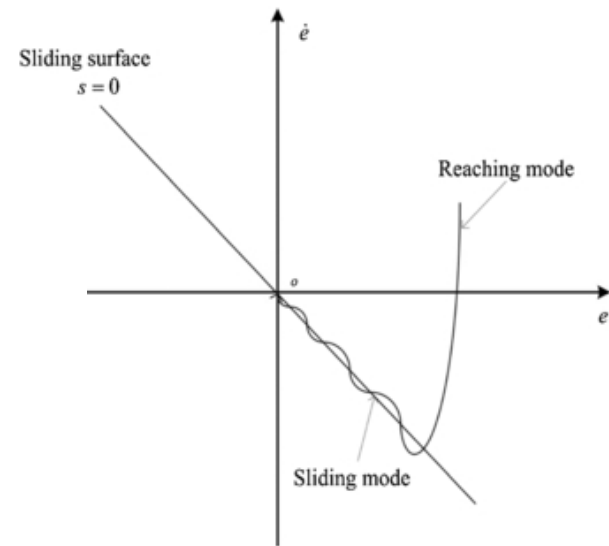
$$s_2 = \dot{y}_e + k_2 y_e + k_3 \theta_e$$

where  $k_1$ ,  $k_2$  and  $k_3$  are positive defined parameters.

According to Gao et al., the dynamics of the sliding surface is the following, which is called the reaching law:

$$\dot{s}_i = -Q_i s_i - P_i \text{sign}(s_i)$$

where  $Q$  and  $P$  are positive defined parameters and its stability can be proven using Lyapunov theorem.





# 4. TWO NON LINEAR CONTROL STRATEGIES

## B. The Sliding Control Approach

Lyapunov candidate function and its time derivative is defined as follows:

$$V = \frac{1}{2} s' s$$

Evaluating its derivative:

$$\dot{V} = s \dot{s}$$

and considering the control law (13), it can be expressed as:

$$\dot{V} = s_1(-Q_1 s_1 - P_1 \operatorname{sgn}(s_1)) + s_2(-Q_2 s_2 - P_2 \operatorname{sgn}(s_2))$$

or alternatively:

$$\dot{V} = -Q_1 s_1^2 - Q_2 s_2^2 - P_1 |s_1| - P_2 |s_2|$$

such that to fulfill the Lyapunov stability theorem,  $Q_1$ ,  $Q_2$ ,  $P_1$  and  $P_2$  have to be semi-positive definite.

Finally, the control law will have the following expression:

$$u_i = u_{eq_i} - u_{c_i}$$

where the first term is called equivalent control and makes the derivative of the sliding surface equal to zero to stay on the sliding surface.

$$u_{c_i} = \frac{Q_i s_i + P_i \operatorname{sgn}(s_i)}{g(x)}$$

$$\dot{s}_1 = 0 :$$

$$u_{eq1} = \dot{v}_r = \frac{-v_r \dot{\theta}_e \sin(\theta_e) - \dot{y}_e \omega_d - y_e \dot{\omega}_d + \dot{v}_d - k_1 \dot{x}_e}{\cos(\theta_e)}$$

$$\dot{s}_2 = 0 :$$

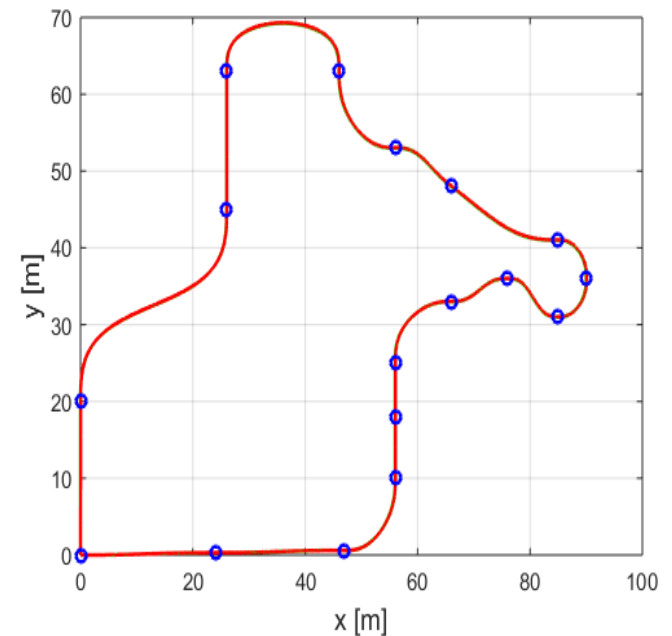
$$u_{eq2} = \omega_r = \omega_d + \frac{-k_2 \dot{y}_e + \dot{w}_d x_e + w_d \dot{x}_e - \dot{v}_c \sin(\theta_e)}{v_r \sin(\theta_e) + k_3}$$

## **4.- Simulation Results**

# 5. SIMULATION RESULTS

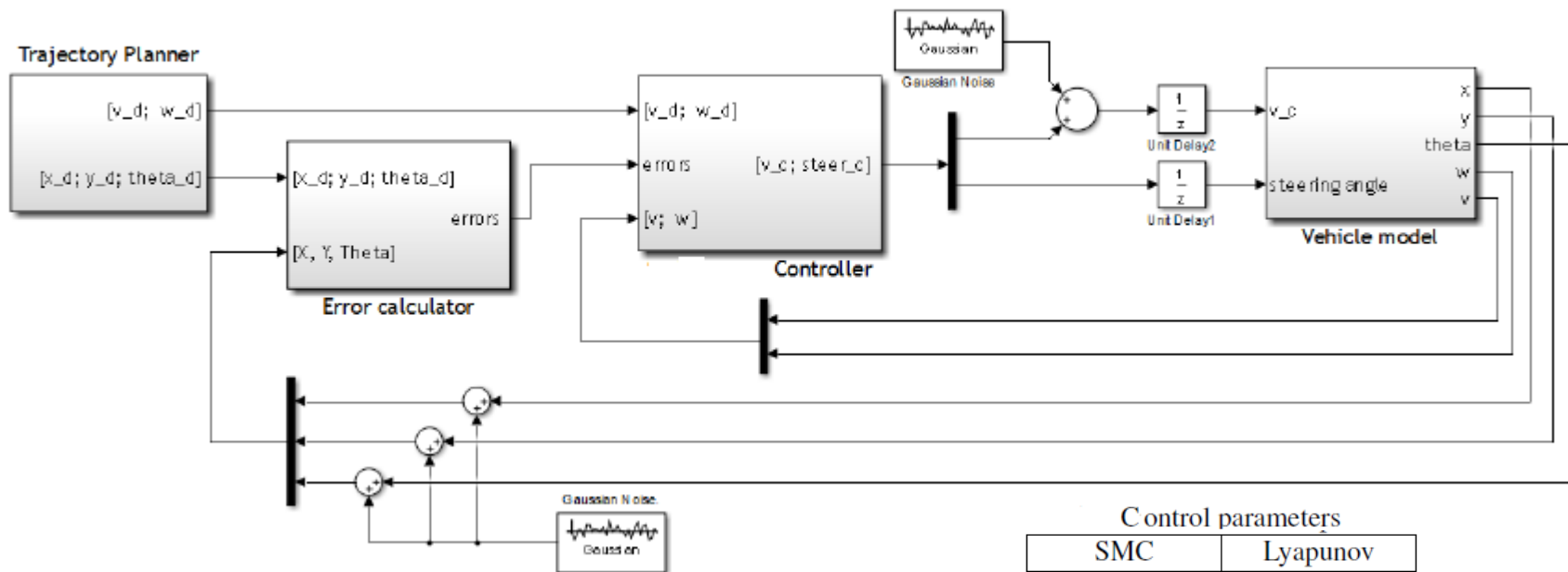
In parallel with the implementation of the controller, a trajectory planner has also been implemented which provides the specific instructions to the control area. The steps to perform the trajectory tracking are:

- 1) The GPS provides to the vehicle a set of forward way points at every segment.
- 2) When a segment finishes, the planner takes the next way point and perform the correct speed profile according to the maximum acceleration allowed. From this segment a set of sub way points.
- 3) Once such a segment has been sampled, at every sample time ( $T_s = 0.1s$ ) the control area takes a sub way point features as a desired configuration and perform the control.



# 5. SIMULATION RESULTS

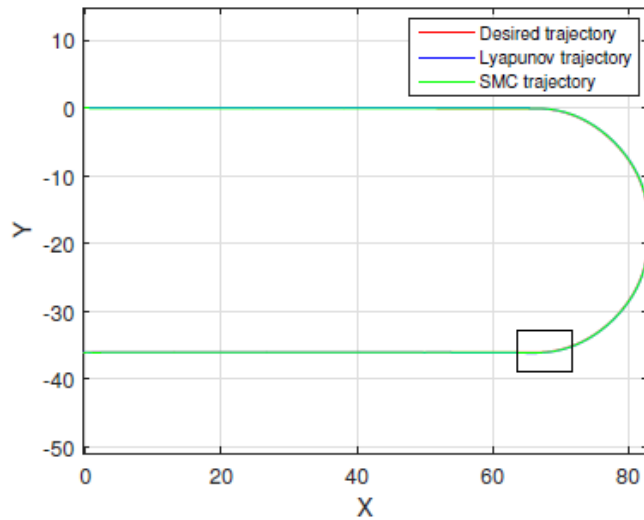
## Matlab/Simulink Simulator



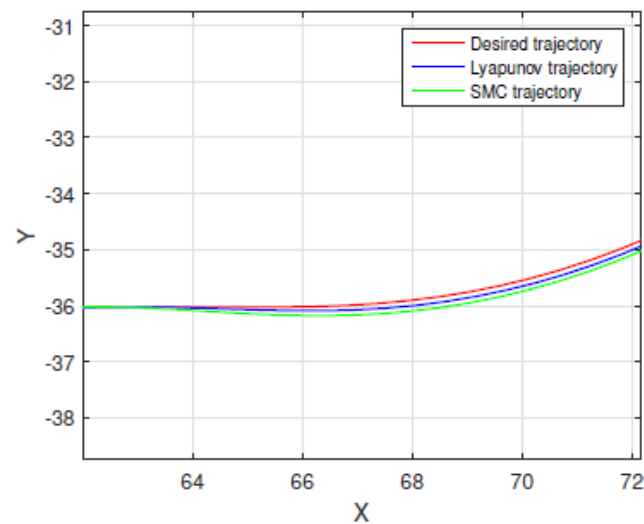
Control parameters

SMC		Lyapunov	
Name	Value	Name	Value
$k_1$	10	$k_1$	10
$k_2$	20	$k_2$	1
$k_3$	25.5	$k_3$	13
$P_1$	0.5		
$Q_1$	0.05		
$P_2$	3.7		
$Q_2$	0.3		

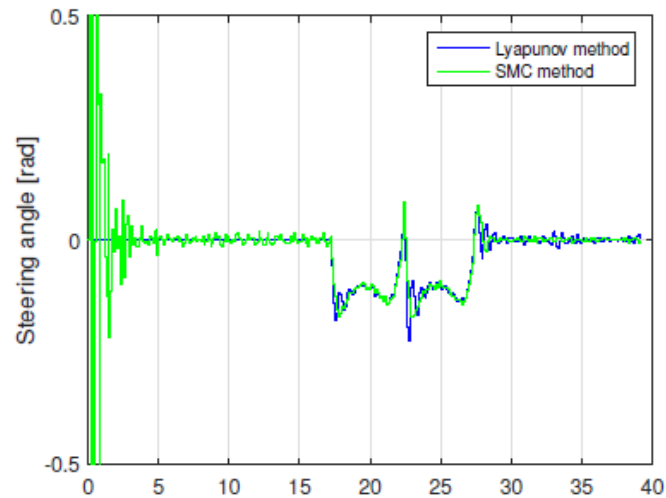
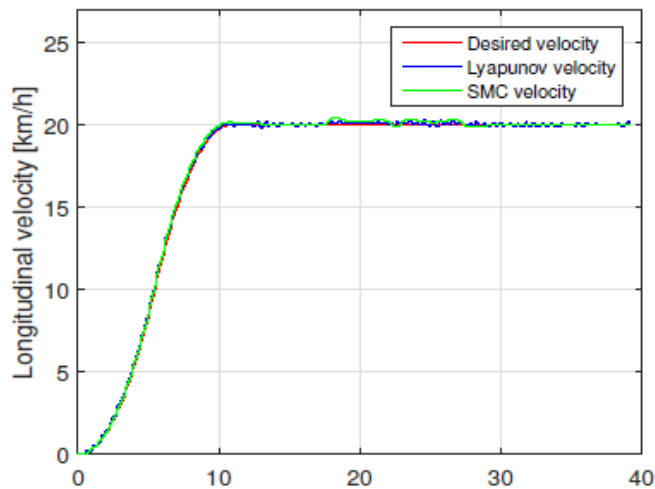
# 5. SIMULATION RESULTS



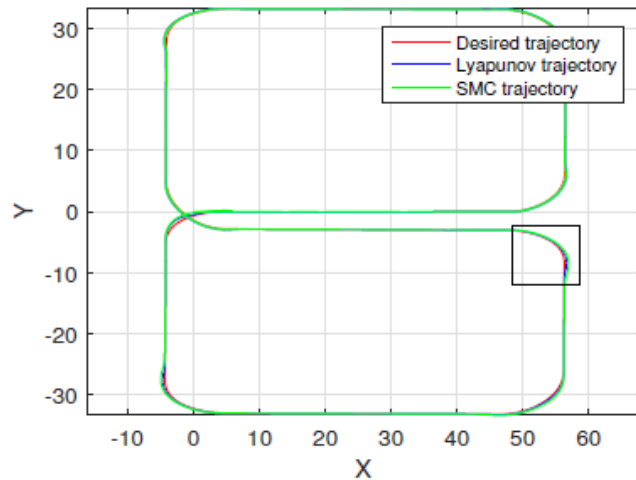
(a)



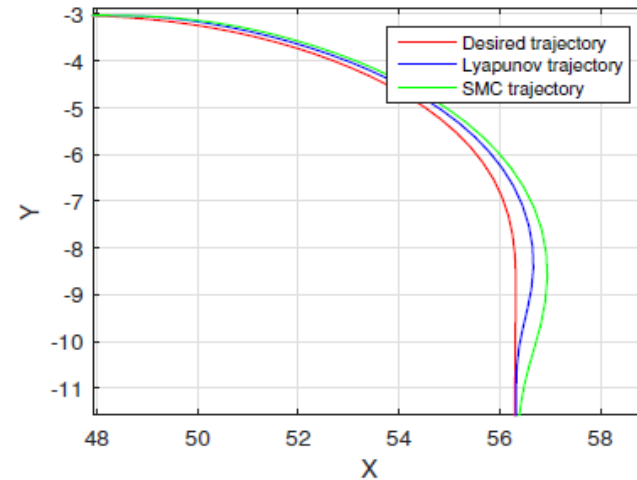
(b)



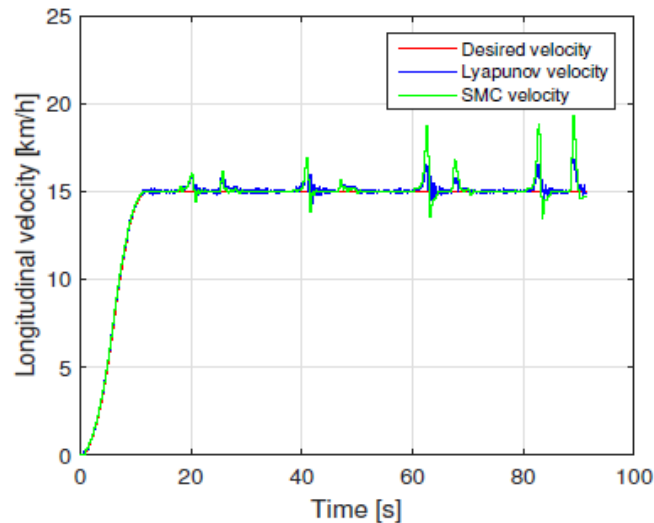
# 5. SIMULATION RESULTS



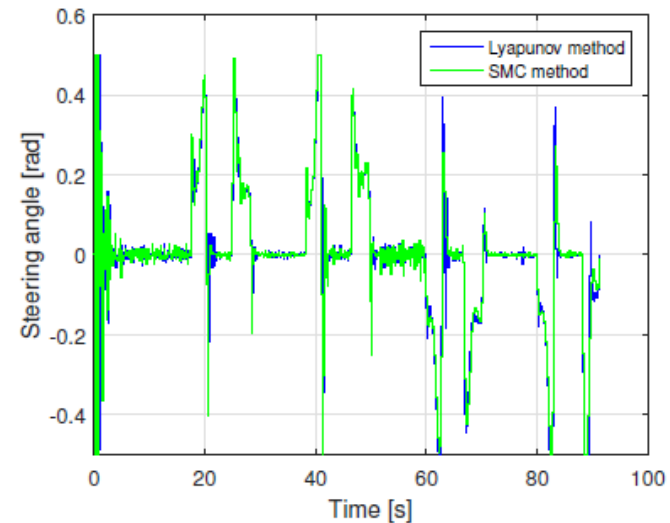
(a)



(b)

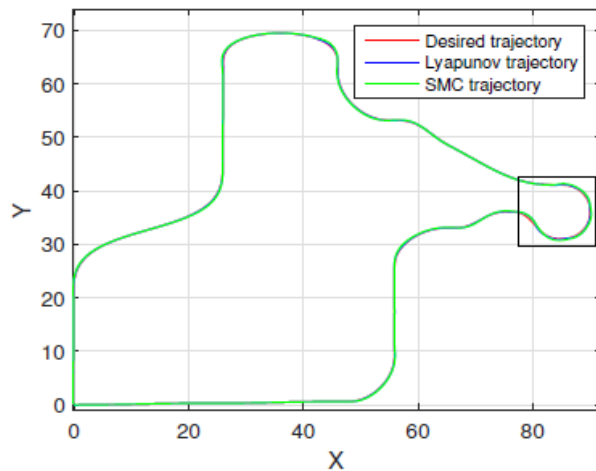


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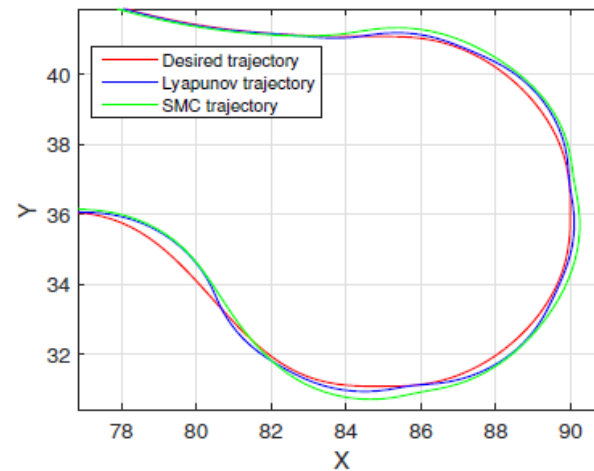


(d)

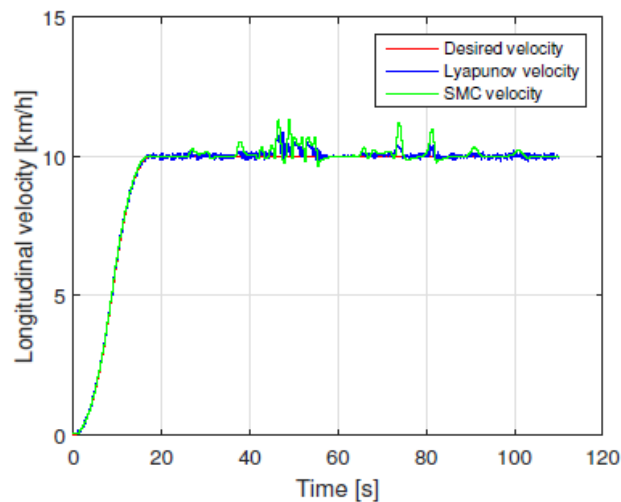
# 5. SIMULATION RESULTS



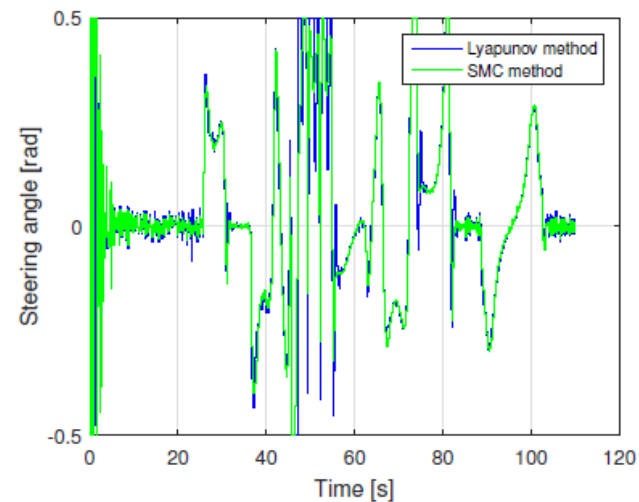
(a)



(b)



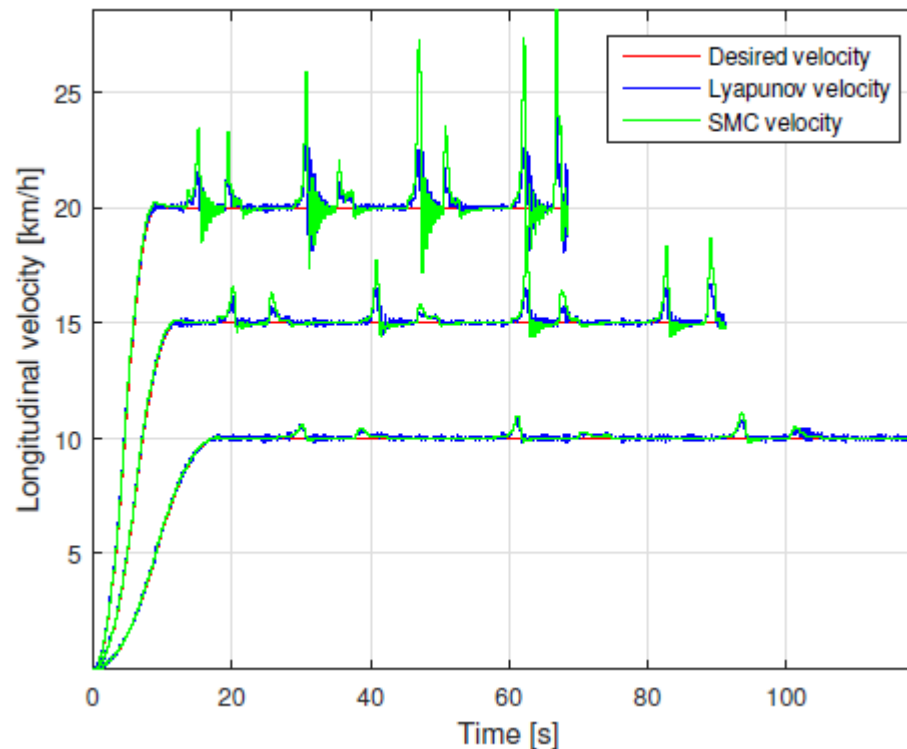
(c)



(d)

# 5. SIMULATION RESULTS

A test in order to find out the limits of robustness of both control techniques has been done. The experiment has consisted in using a set of velocity scenarios over the same circuit and with the same initial control parameters.





# 5. SIMULATION RESULTS

It can be appreciated how the SMC method computes stronger velocity control actions than Lyapunov technique under situations of higher velocity. Accordingly, it can be seen how the Lyapunov control algorithm achieves less longitudinal error than SMC algorithm in curves. In straight segments, the SMC method reaches a null longitudinal error while Lyapunov method presents a steady state error.

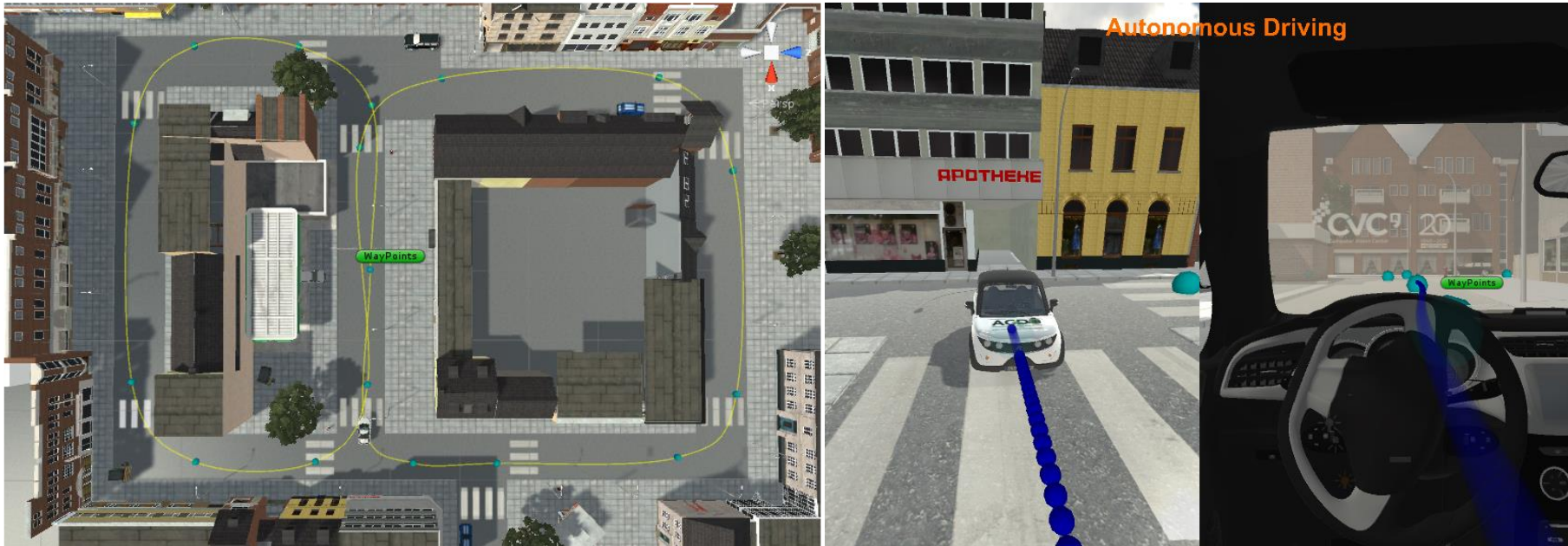
With respect to the steering angle control action, the SMC technique computes the best actions, in spite of the first five seconds behaviour which can be eliminated as we discussed in the circuits test conclusions. Lyapunov steering angle control action performs an oscillation when it tries to stabilise to zero degrees.

Regarding lateral and orientation error, the SMC technique has a faster mitigation of the error but due to this it performs higher errors.

# 5. SIMULATION RESULTS

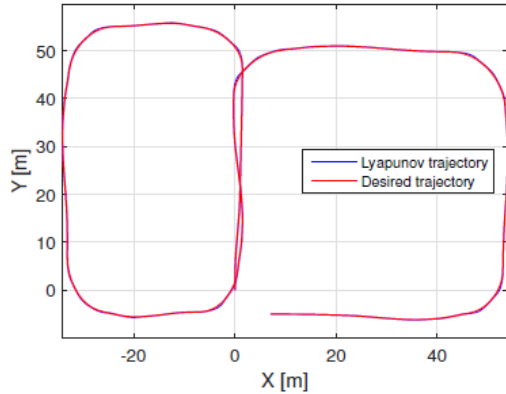
## Unity Simulator

The model of the vehicle is a complete dynamic model which takes into account the suspension dynamics, the drivetrain dynamics and even the engine dynamics among others. The developed circuit, for testing the control techniques, corresponds to the already presented circuit

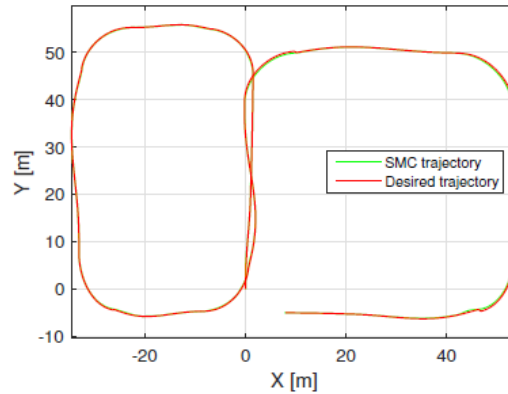


# 5. SIMULATION RESULTS

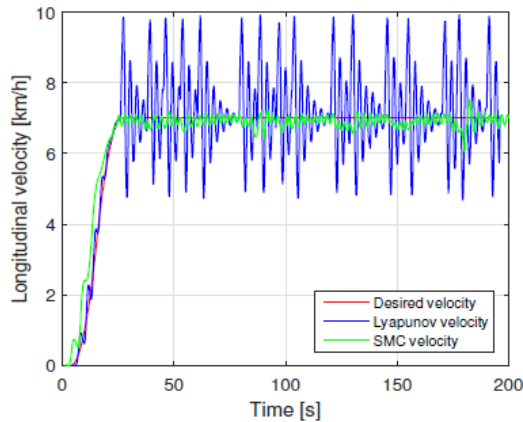
## Unity Simulator



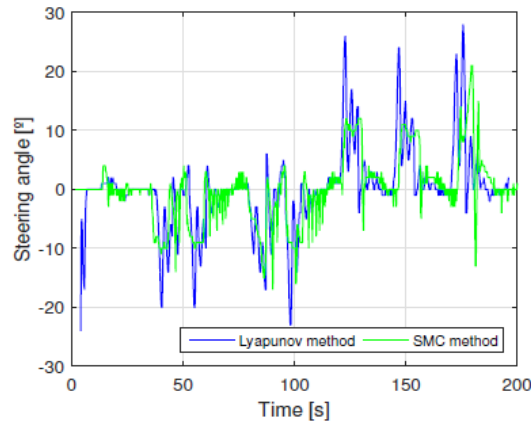
(a)



(b)



(c)



(d)

SMC		Lyapunov	
Name	Value	Name	Value
$k_1$	0.15	$k_1$	1.5
$k_2$	5	$k_2$	1.6
$k_3$	7	$k_3$	0.7
$P_1$	0.3		
$Q_1$	0.05		
$P_2$	0.75		
$Q_2$	0.07		

## 6.Experimental Results

# 6. EXPERIMENTAL RESULTS

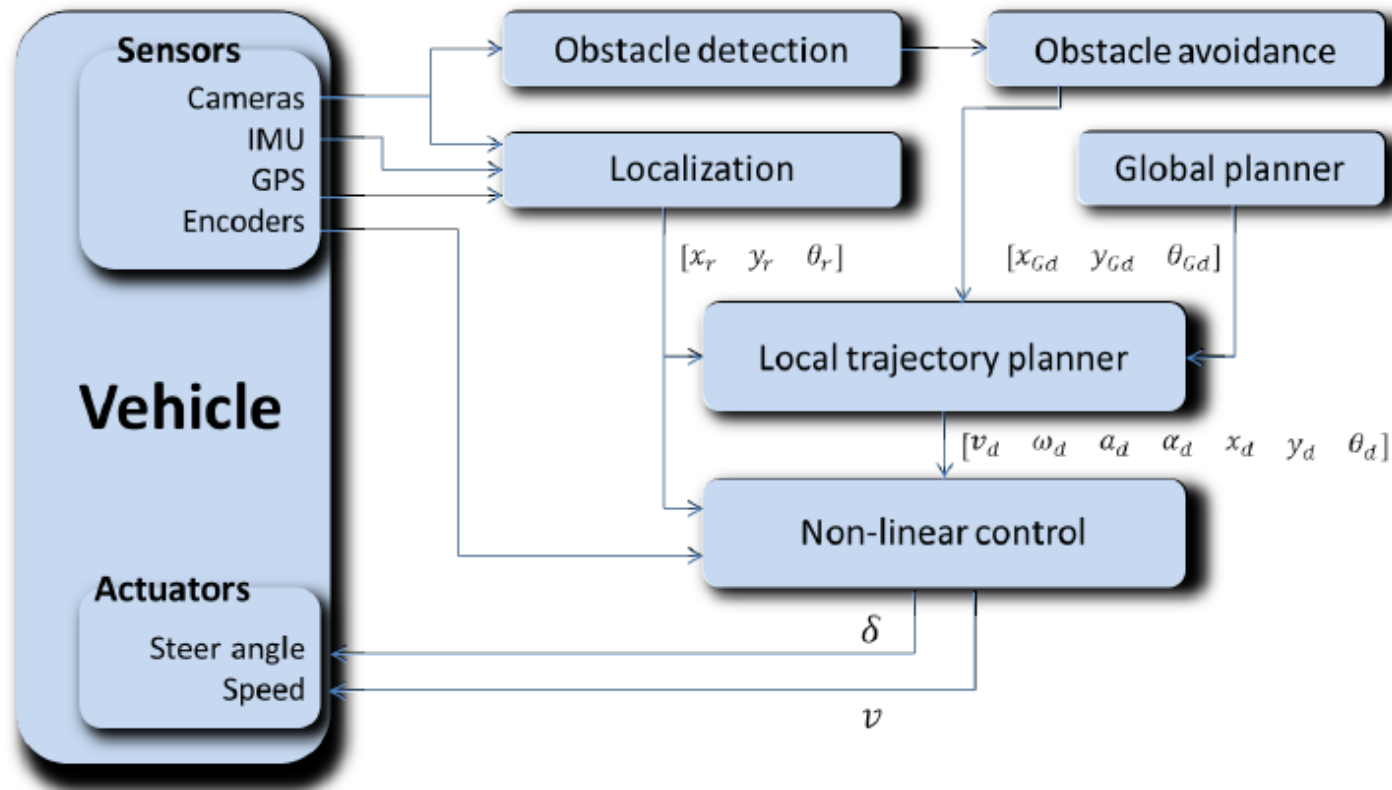
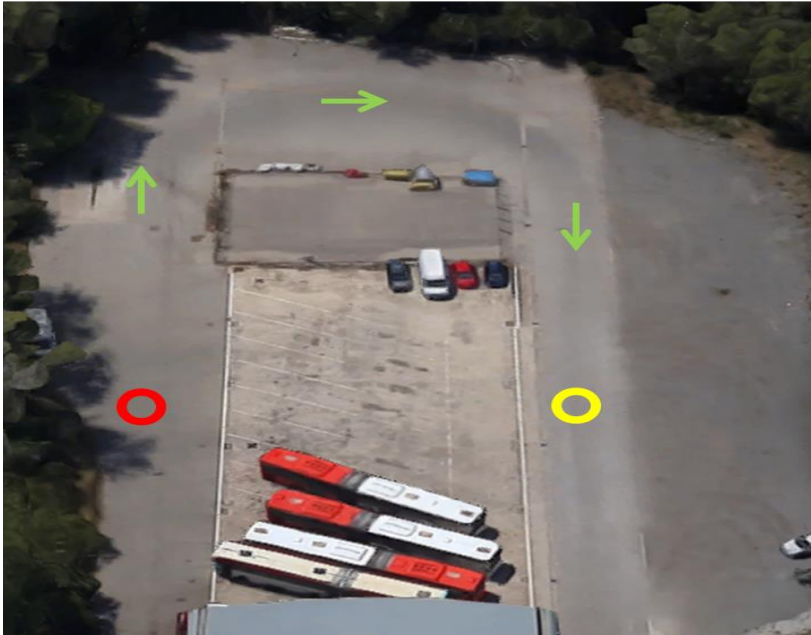


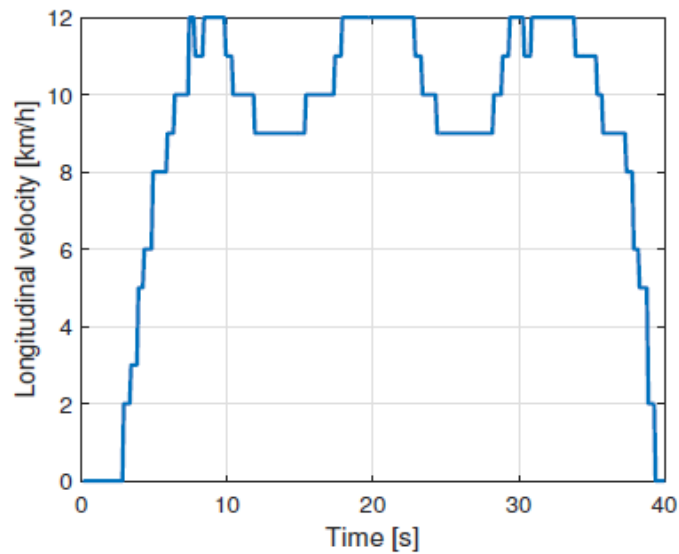
Figure : Architecture diagram of the real vehicle where main variables are showed

# 6. EXPERIMENTAL RESULTS

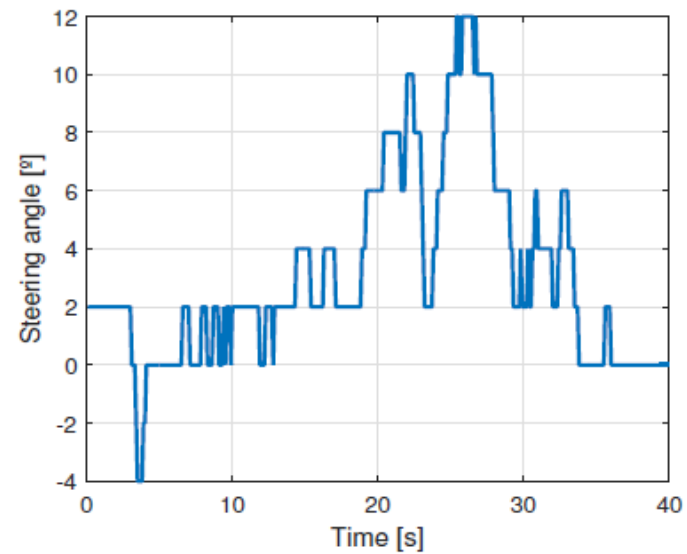


Control parameters used in the real test

Lyapunov	
Name	Value
$k_1$	1.5
$k_2$	1.6
$k_3$	0.7



(a)



(b)

## 7. Conclusions

# 4. CONCLUSIONS

- Both control methods have demonstrated to be robust with respect to some noise and disturbances, and the obtained results show the effectiveness of such proposed control schemes.
- Both control strategies have been already tested on a virtual reality simulation developed in Unity simulator and the SMC approach works better than Lyapunov control.
- The first real tests are being tested in a real car available at the Computer Vision Center and the results show promised performances.
- In next future other techniques of multivariable control such as MPC will be implemented and compared with the two other approaches in more complex real and simulated scenarios.